

12.
$$\begin{aligned} x - 2y + 2z &= 3 \\ 2x + 3y - z &= 5 \\ y + z &= 5 \end{aligned} \rightarrow \begin{pmatrix} 1 & -2 & 2 & | & 3 \\ 2 & 3 & -1 & | & 5 \\ 0 & 1 & 1 & | & 5 \end{pmatrix}$$

$R_2 - 2R_1 \rightarrow R_2$ (Multiply 1 row by 2 row and subtract it from 2 row)

$$\begin{pmatrix} 1 & -2 & 2 & | & 3 \\ 0 & 7 & -5 & | & -1 \\ 0 & 1 & 1 & | & 5 \end{pmatrix} \rightarrow R_2 / 7 - R_2 \text{ (divide the 2 row by 7)}$$

$R_1 + 2R_2 \rightarrow R_1$ (Multiply 2 row by 2 and add it to 1 row);

$R_3 - 1R_2 \rightarrow R_3$ (Multiply 2 row by 1 and subtract it from 3 row)

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{4}{7} & | & \frac{19}{7} \\ 0 & 1 & -\frac{5}{7} & | & -\frac{1}{7} \\ 0 & 0 & \frac{12}{7} & | & \frac{36}{7} \end{pmatrix}$$

$R_3 / \frac{12}{7} \rightarrow R_3$ (divide 3 row by $\frac{12}{7}$)

$$\begin{pmatrix} 1 & 0 & \frac{4}{7} & | & \frac{19}{7} \\ 0 & 1 & -\frac{5}{7} & | & -\frac{1}{7} \\ 0 & 0 & 1 & | & 3 \end{pmatrix}$$

$R_1 - \frac{4}{7} R_3 \rightarrow R_1$ (Multiply 3 row by $\frac{4}{7}$ and subtract it from 1 row);
 $R_2 + \frac{5}{7} R_3 \rightarrow R_2$ (Multiply 3 row by $\frac{5}{7}$ and add it to 2 row)

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right) \rightarrow \begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$$

Question 13 Using Matrix and Inverse.

$$2x + y = -1$$

$$3x + y = -3$$

soln.

$$AX = B$$

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}^{-1} = \frac{1}{2 \times 1 - 1 \times 3} \times \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$$

$$X = A^{-1} \times B$$

$$= \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\text{Therefore } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$x = -2 \\ y = 3$$

Question 14

Inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

Solution

(1) Augmented Matrix.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array} \right)$$

find pivot of first column in the 1st row

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array} \right)$$

We eliminate the 1st column.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 & 1 \end{array} \right)$$

→ find pivot of 2nd column & row.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 & 1 \end{array} \right)$$

Eliminate the 2nd column.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & 10 & -3 & 1 \end{array} \right)$$

→ find the pivot of 3rd column & row

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & 10 & -3 & 1 \end{array} \right)$$

→ The inverse of the matrix is on the right

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & 10 & -3 & 1 \end{array} \right)$$

$$\underline{\underline{A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -3 & 1 \end{pmatrix}}}$$

Question 15.

Determinant of a Matrix

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$|A| = 1(5 \times 9 - 6 \times 8) - 4(2 \times 9 - 3 \times 8) + 7(2 \times 6 - 3 \times 5)$$

$$\Rightarrow 1(45 - 48) - 4(18 - 24) + 7(12 - 15)$$

$$\Rightarrow 1(-3) - 4(-6) + 7(-3)$$

$$\Rightarrow -3 + 24 - 21$$

$$\Rightarrow \underline{\underline{0}}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & 6 & -9 \end{pmatrix} = A^{-1}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & 6 & -9 \end{bmatrix} = A^{-1}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & 6 & -9 \end{pmatrix}$$

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